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NON-SYNCHRONOUS TILTING PAD BEARING CHARACTERISTICS

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ABSTRACT

The characteristics of fluid film bearings are speed-dependent. Recently it is discussed more and more, that the characteristics may also be frequency dependent, especially in case of tilting pad bearings.

In the present paper a model for tilting pad bearings including the pad inertias and coordinates for the pad rotational angles and their rotational velocities is created. Dynamic bearing coefficients are calculated including these coordinates by solving the Reynolds equation for the perturbed equilibrium position. The perturbation includes the rotor displacements and velocities as well as the pad tilting angles and angular velocities. Thus a system of linear 2+Np differential equations with Np as the number of pads is created for the bearing. From these equations transfer functions for the bearing force / rotor displacement relation can be determined. These transfer functions clearly yield frequency dependent stiffness and damping coefficients, although all coefficients of the complete system with 2+Np equations are only speed dependent.

The tilting pad bearing model with the 2+Np equations is included into a rotor model by means of the rotor dynamic program MADYN 2000 [1]. The rotor dynamic stability of the rotor considering the non-synchronous behaviour of the tilting pad bearings is presented for different bearing geometries showing cases with strong and weak frequency dependence.

INTRODUCTION

Fluid film bearing forces on the rotor are normally modeled by stiffness and damping matrices and sometimes with mass matrices as shown in the following equation, which includes the mass term:

$$\begin{bmatrix}-F_2\\-F_3\end{bmatrix}=\begin{bmatrix}k_{22} & k_{23}\\k_{32} & k_{33}\end{bmatrix}\begin{bmatrix}x_2\\x_3\end{bmatrix}+\begin{bmatrix}d_{22} & d_{23}\\d_{32} & d_{33}\end{bmatrix}\begin{bmatrix}\dot{x}_2\\\dot{x}_3\end{bmatrix}+\begin{bmatrix}m_{22} & m_{23}\\m_{32} & m_{33}\end{bmatrix}\begin{bmatrix}\ddot{x}_2\\\ddot{x}_3\end{bmatrix} \quad (1)$$

The discussion about the frequency dependence of the rotor dynamic coefficients of tilting pad bearings is controversial. A good overview of the research in this area is given in the very recent paper of C.R. Carter and D.W. Childs [2].

In the same paper measurements are compared to calculated results. The main conclusion is that a bearing model including stiffness, damping and mass coefficients yields good agreement to measured results without considering any frequency dependence.

In contrast to [2] the bearing forces in [3] are formulated with stiffness and damping coefficients only, i.e. without a mass term. An adaptation of the forces to measurements clearly yields frequency dependent coefficients. The stiffness decreases with increasing frequency, whereas the damping increases.

In the model presented here the bearing forces are formulated in a more general way than in equation (1) including coordinates for the tilting of the pads and considering the pads moment of inertia, whereas in the bearing models generally used nowadays the coordinates for the pad tilting are eliminated. In [4] is shown that this implies neglecting tangential pad forces which is true if the pad immediately follows any shaft movement, i.e. the pad inertia and the inhibition of the pad tilting by the fluid film damping are neglected. The resulting stiffness and damping coefficients for the lateral journal coordinates in [4] are purely speed dependent.

The starting point for the analysis of the bearing forces of the extended model presented here is the Reynolds equation, as in all common models. The viscosity is assumed as constant. In a first step the Reynolds equation is solved to receive the static equilibrium of the rotor journal and each pad due to the static forces. In a second step the equilibrium is perturbed. The perturbation includes the rotor displacements and velocities as well as the pad tilting angles and angular velocities.

NOMENCLATURE

d	damping coefficient
e	journal excentricity
h	oil film thickness
k	stiffness coefficient
m	mass coefficient
m	$1-\frac{C_b}{C_p}$, preload
p	pressure
t	time

t_p pad thickness
x displacement coordinate
z axial direction
C_p R_p-R_j, pad clearance
C_b R_b-R_j, bearing clearance
F force
I_p pad moment of inertia
L bearing width
R_b bearing radius
R_j rotor journal radius
R_p pad radius
U $\Omega * R_j$, circumferential speed
D damping matrix
G gyroscopic matrix
K stiffness matrix
M mass matrix
X vector of coordinatess
ε $\frac{e}{C_b}$, dimensionless exctricity
μ oil viscosity
ω frequency in [rad/s]
Θ circumferential angle
Ψ pad tilting angle
Ω rotor speed in [rad/s]

Indices
2 2-direction
3 3-direction
R index for rotor

MATHEMATICAL MODEL

Bearing force analysis of tilting pad bearings

The general geometry of one pad of a tilting pad bearing is shown in figure 1.

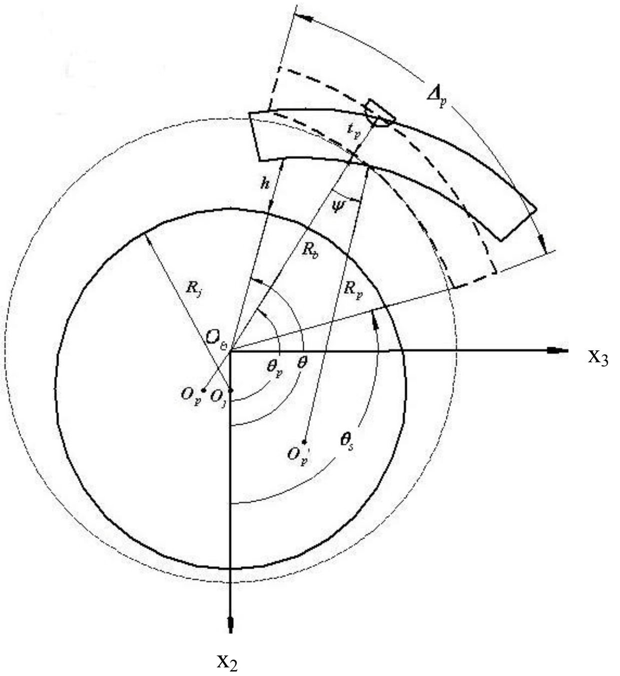

Fig.1 Pad geometry of a tilting pad bearing

Using the following dimensionless parameter

$$z = L\bar{z},\ p = p^{*}\bar{p},\ h = C_b\bar{h},\ t = \frac{\bar{t}}{\Omega} \quad (1)$$

with the reference pressure

$$p^{*} = \frac{6U\mu R_j}{{C_b}^2} \quad (2)$$

The Reynolds equation can be written as follows:

$$\frac{\partial}{\partial\theta}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\theta}\right) + \upsilon^2\frac{\partial}{\partial\bar{z}}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\bar{z}}\right) = \frac{\partial\bar{h}}{\partial\theta} + 2\frac{\partial\bar{h}}{\partial\bar{t}} \quad (3)$$

Further in the paper we will omit the bar above the variables, working only with dimensionless variables.

For the oil film thickness the following equation applies:

$$h(\theta,z) = \frac{1}{1-m^i} - \varepsilon_0\cos\theta - x_2\cos\theta - x_3\sin\theta - \frac{m^i}{1-m^i}\cos\left(\theta-\theta_p^i\right) - \psi^i\chi^i\sin\left(\theta-\theta_p^i\right) \quad (4)$$

with

$$\chi = \frac{R_p^i + t_p^i}{C_b} \quad (5)$$

The oil film thickness can be presented as the sum of the thickness in equilibrium position and an additional perturbation term:

$$h = h_0 + \delta h \quad (6)$$

with

$$h_0 = \frac{1}{1-m^i} - \varepsilon_0\cos\theta - \frac{m^i}{1-m^i}\cos\left(\theta-\theta_p^i\right) - \psi_0^i\chi^i\sin\left(\theta-\theta_p^i\right) \quad (7)$$

and

$$\delta h = -x_2 cos\theta - x_3 sin\theta - \delta\psi^i\ \psi^i \sin\left(\theta - \theta_p^i\right) \quad (8)$$

For a small perturbation δh a similar decomposition can be made for the pressure.

$$p = p_0 + \delta p \quad (9)$$

By substituting (9) into (3), neglecting terms of second and higher order and using (7) and (8), we can write equations for p_0 and δp. The structure of the right hand side in the equation for δp allows to write the perturbation pressure term δp in the following form:

$$\delta p(t,\theta,z) = p_{x_2}(\theta,z)x_2 + p_{x_3}(\theta,z)x_3 + p_{1x_2}(\theta,z)\dot{x}_2 + p_{1x_3}(\theta,z)\dot{x}_3 + \chi^i\left(p_{\delta\psi}(\theta,z)\delta\psi^i + p_{1\delta\psi}(\theta,z)\delta\dot{\psi}^i\right) \quad (10)$$

Thus we receive a set of equations for separate components of δp from which the dynamic bearing force can be written in the following form:

$$\mathbf{C}\mathbf{X} + \mathbf{D}\dot{\mathbf{X}} + \mathbf{E}\boldsymbol{\Psi} + \mathbf{H}\dot{\boldsymbol{\Psi}} = \mathbf{F} \qquad (11)$$

$$\mathbf{I}_p\ddot{\boldsymbol{\psi}} + \tilde{\mathbf{C}}\mathbf{X} + \tilde{\mathbf{D}}\dot{\mathbf{X}} + \tilde{\mathbf{E}}\boldsymbol{\psi} + \tilde{\mathbf{H}}\dot{\boldsymbol{\psi}} = \mathbf{0} \qquad (12)$$

The vectors and matrices in (11) and (12) are:
$\mathbf{X}$: Vector of the lateral journal positions (coordinates x_2, x_3).
$\boldsymbol{\Psi}$: Vector of the tilting angles of each pad.
$\mathbf{F}$: Vector of bearing forces in 2- and 3-direction.
$\mathbf{I}_p$: Diagonal matrix with the moments of inertias of each pad
$\mathbf{C}, \mathbf{D}, \mathbf{E}, \mathbf{H}, \tilde{\mathbf{C}}, \tilde{\mathbf{D}}, \tilde{\mathbf{E}}, \tilde{\mathbf{H}}$: Matrices with speed dependent damping and stiffness coefficients.

The coefficients in equations (11) and (12) are only speed dependent, not frequency dependent.

The transfer functions between bearing forces and lateral journal coordinates in equation (13) can be gained by assuming harmonic forces, displacements and tilting angles with the same frequency and eliminating the tilting angles.

$$\hat{\mathrm{F}} = \left\{(\mathrm{i}\mathbf{D}\omega + \mathbf{C}) - (\mathrm{i}\mathbf{H}\omega + \mathbf{E})\left[\left(\mathrm{I}_P\,\omega^2 + \mathrm{i}\tilde{\mathbf{H}}\omega + \tilde{\mathbf{E}}\right)^{-1}\left(\mathrm{i}\tilde{\mathbf{D}}\omega + \tilde{\mathrm{C}}\right)\right]\right\}\hat{\mathrm{X}} \qquad (13)$$

$\hat{\mathbf{F}}, \hat{\mathbf{X}}$ contain the complex amplitudes in 2- and 3-direction. It can be seen, that equation (13) is a relation, which cannot be simply expressed by constant mass, damping and stiffness coefficients in the form of equation (14).

$$-\hat{\mathbf{F}} = (-\omega^2\mathbf{M} + \mathrm{i}\omega\mathbf{D} + \mathbf{K})\hat{\mathbf{X}}. \qquad (14)$$

Combining the structure and the tilting pad bearing

The equation of motion for the rotor, which can also contains the supports and other bearings than tilting pad bearings, is as follows:

$$\mathbf{M}\ddot{\mathbf{X}}_R + (\mathbf{D} + \mathbf{G})\dot{\mathbf{X}}_R + \mathbf{K}\mathbf{X}_R = \mathbf{F} \qquad (15)$$

The vector $\mathbf{X}_R$ contains all structural coordinates of the rotor and of the bearing supports. The vector $\mathbf{F}$ contains the forces on the rotor and the support, among others the bearing forces of the tilting pad bearings. Substituting equation (11), which is coupled to equation (12), into equation (15), yields the equation of the combined system.

Equations (11) and (12) can be rearranged to receive a state space form, which can be used to model the tilting pad bearing in a similar way as a magnetic bearing in MADYN 2000 [1].

EXAMPLE

Bearings

The bearing characteristics of the extended model are shown for three different tilting pad bearings:

(1) A 4-tilting pad bearing with 55% offset
(2) A 5-tilting pad bearing with 55% offset
(3) A 4-tilting pad bearing with 60% offset

The load in all three examples is oriented between the pads. The bearings are rather lightly loaded: The specific bearing load is 5bar. The geometry of the three bearings is shown in figures 1 to 3.

Oil with a viscosity according to ISO VG46 with an inlet temperature of 40°C is used. The mean temperature at a speed of 6000rpm according to a DIN analysis (see [5]) is 55°C.

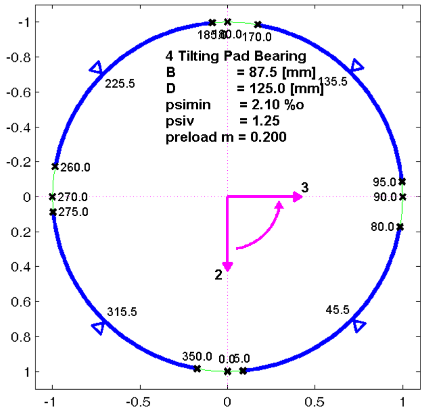

Fig.1 4-tilting pad bearing, 55% offset

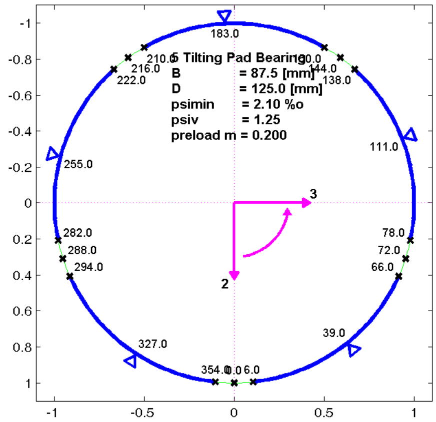

Fig.2 5-tilting pad bearing, 55% offset

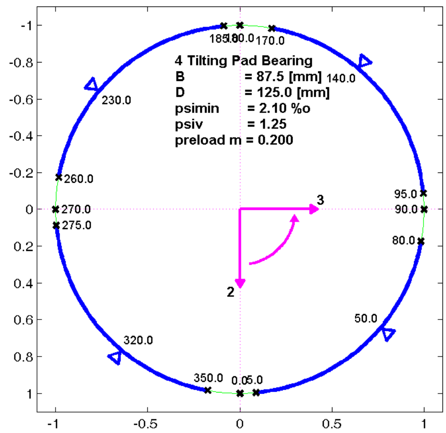

Fig.3 4-tilting pad bearing, 60% offset

Rotor

The effect of the bearings on a rotor are studied with the example rotor in figure 4.

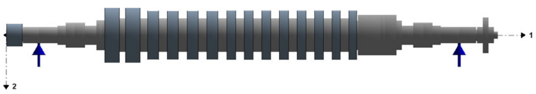

Fig.4 Example rotor

The rotor has a maximum speed of 6000rpm or 100rps. It is rather slim and has a high ratio of the maximum speed to the first natural frequency (so called flexi ratio) of about 3.

RESULTS

Bearing Characteristics

The frequency dependent stiffness and damping coefficients for lateral coordinates of the three bearings at a speed of 6000rpm or 100Hz are shown in figures 5 to 7. The coefficients are derived from the transfer functions (equation (13)) as follows:

$$k_{i,j} = Re(\frac{F_{i,j}}{x_{i,j}}) \quad (16)$$

$$d_{i,j} = \frac{Im(\frac{F_{i,j}}{x_{i,j}})}{\omega} \quad (17)$$

where i,j are indices for the two directions 2,3.

The synchronous coefficients in the figures are marked. They agree exactly with the classically calculated purely speed dependent damping and stiffness coefficients for lateral coordinates according to the DIN standard [5].

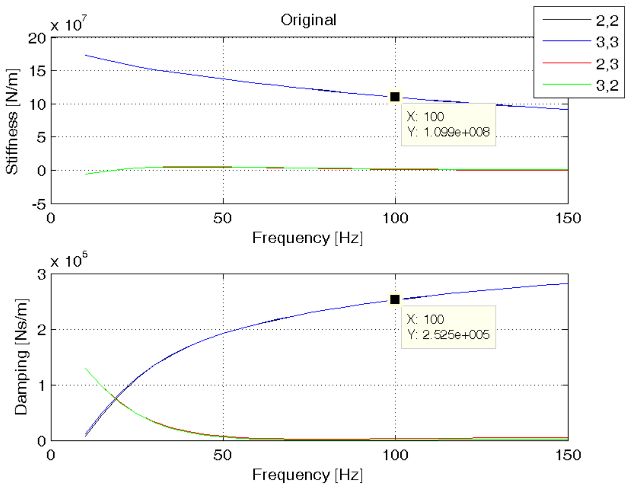

Fig.5 Stiffness and damping coefficients at 6000rpm, 4-tilting pad bearing 55% offset

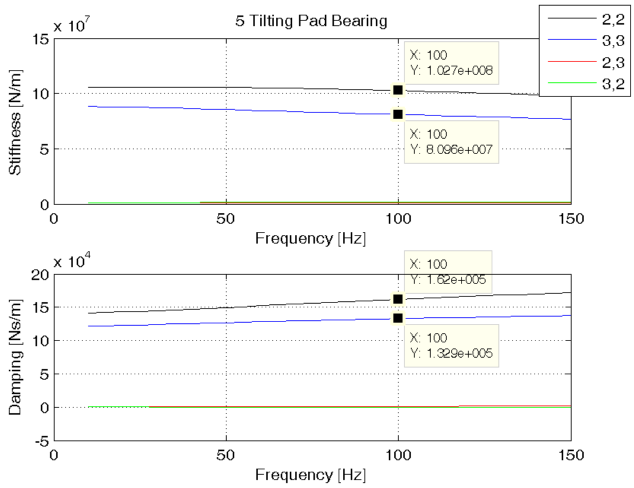

Fig.6 Stiffness and damping coefficients at 6000rpm, 5-tilting pad bearing 55% offset

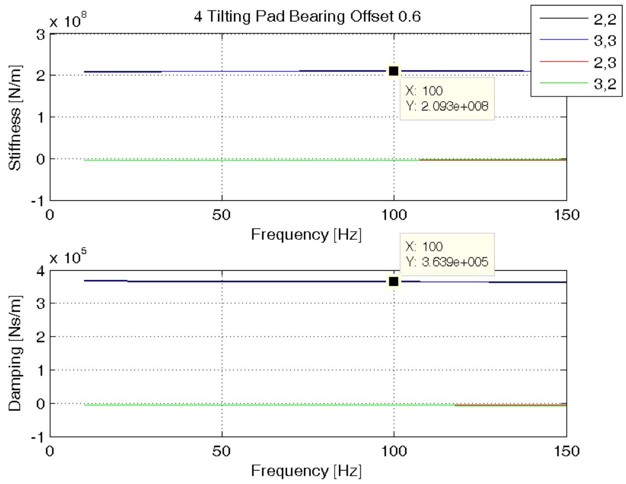

Fig.7 Stiffness and damping coefficients at 6000rpm, 5-tilting pad bearing 60% offset

For the 4-tilting pad bearing with 55% offset the coefficients are strongly frequency dependent. The stiffness increases with decreasing frequency (i.e. in the sub-synchronous range) and the damping decreases. For the damping of the first natural mode of a rotor, which is normally in the sub-synchronous frequency range, this has an adverse effect. This trend is in accordance to the measured results in [3] of a 5-tilting pad bearing with central pivot in load on pad as well as load between pad configurations.

The frequency dependence is lower in case of the 5-tilting pad bearing with 55% offset. This is most probably due to the shorter pads.

Increasing the offset of the 4-tilting pad bearing to 60% practically eliminates the frequency dependence. The stiffness as well as damping coefficient considerably increase compared to 55% offset, which is an unwanted effect in case of flexible rotors, since this normally reduces the system damping of the first natural mode.

The model presented here yields cross coupling coefficients. However, they are considerably smaller than the direct coefficients, except in case of the damping of the 4-tilting pad bearing at very low frequencies.

Neglecting the pad inertia practically yields the same results. This means the frequency dependence of the 4-tilting pad bearing with 55% offset is not mainly caused by the inertia but by the inhibition of the pad tilting by damping.

Natural Modes of the rotor bearing system at maximum speed

The first forward whirling natural mode of the rotor at 6000rpm with the synchronous coefficients in figure 5 has a frequency of 31.62Hz and a damping ratio of 3.7% (log.dec. 23.3%). It is shown in figure 8.

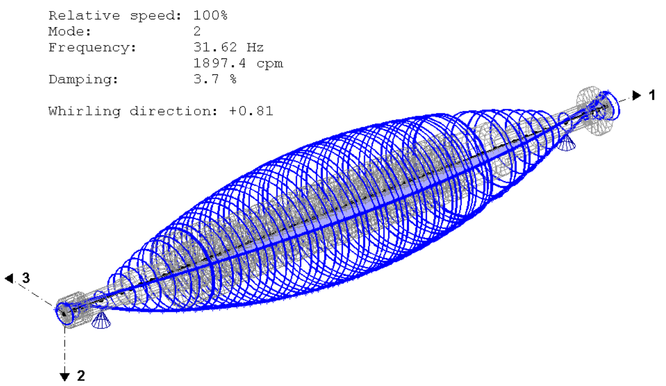

Fig.8 First natural mode with synchronous coefficients, 4-tilting pad bearings, 55% offset

The eigenvalues of the 1st backward whirling (-) and forward whirling (+) bending mode of the rotor on 4-tilting pad bearings are shown in figure 9 for two models of the tilting pad bearings: The classic model with synchronous purely speed dependent coefficients (black) and the extended model (red). It can be clearly seen, that due to the higher stiffness and lower damping of the bearing in the sub-synchronous frequency range the frequencies slightly increase and the system damping ratio considerably decreases.

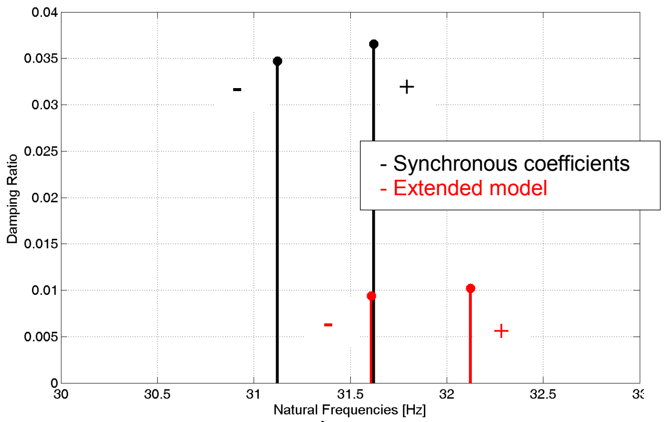

Fig.9 Eigenvalues of the 1st mode, 4-tilting pad bearing, 55% offset

The eigenvalues of the system with the different bearing types calculated with the extended model are shown in figure 10. The 5-tilting pad bearing yields the best damping, because it has the lowest stiffness. The 4-tilting pad bearing with 60% offset yields a lower frequency, in spite of the higher stiffness, which can be attributed to the higher damping coefficient. The damping ratio is low due to the high bearing stiffness.

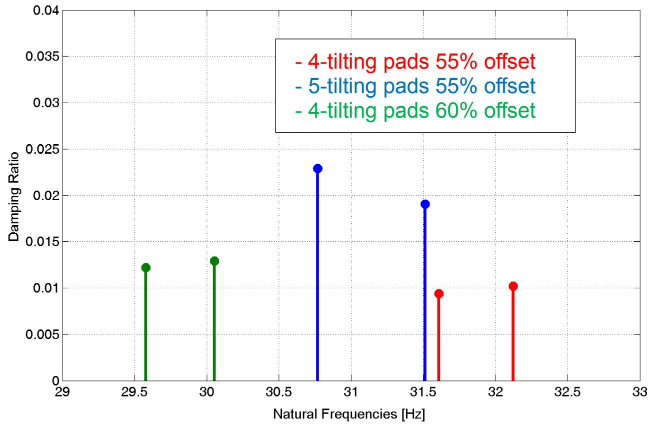

Fig.10 Eigenvalues of the 1st mode, extended model

CONCLUSIONS

Tilting pad bearing characteristics with an extended model including the tilting angle of the pads are calculated. From the model the transfer function of the bearing force / lateral rotor displacement relation is derived. The transfer function can also be expressed as frequency dependent stiffness and damping coefficients. Using additional mass coefficients would not yield frequency independent coefficients. The extended bearing model is combined with the rotor model in the rotor dynamic software MADYN 2000.

The characteristics of 3 different tilting pad bearings with load on pad arrangement are calculated at the nominal speed of a corresponding rotor. A 4-tilting pad bearing with 55% offset yields strongly frequency dependent stiffness and damping coefficients; the stiffness increases and the damping decreases in the sub-synchronous frequency range. Other researchers have found a similar tendency (see [3]). A 5-tilting pad bearing with the same offset yields less frequency dependent stiffness and damping coefficients and a 4-tilting pad bearing with an offset of 60% practically has frequency independent stiffness and damping coefficients. The synchronous coefficients in all cases are similar to classically calculated coefficients, where the tilting angles are eliminated.

Applying the extended bearing model to a rather flexible rotor with a flexi ratio of about 3 can yield considerably lower damping ratios of the 1st natural mode than using the classical model. For the case of the 4-tilting pad bearing the damping ratio drops to less than one third.

The extended model is not yet experimentally validated. However, the following facts confirm it to some extent:

(1) Other researchers have found similar trends.
(2) The synchronous coefficients are identical to classically calculated coefficients.
(3) From the experience with some rotors on tilting pad bearings with a small or no offset can be concluded that

the damping is much lower than an analysis with classic frequency independent coefficients suggests.

Bearings with a small offset are sometimes used, because they analytically yield a lower stiffness and thus a higher system damping ratio of the first bending mode of flexible rotors. The present results of the extended model do not confirm this behavior gained with classical models.

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